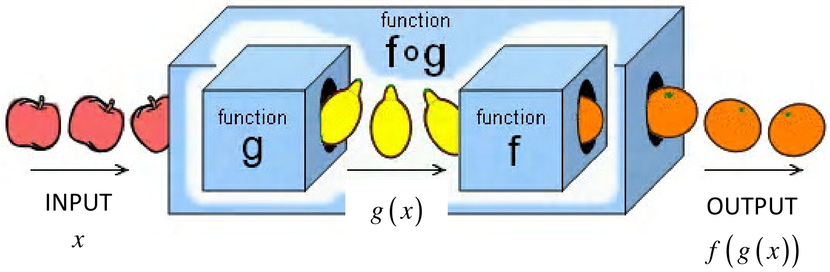
Composite Functions

Composite Function: In a composite function, one function is performed, and then a second function is performed on the result of the first function. $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$.



Hints:

- Work inside out. Plug the input into the inside function, then plug the result into the outside function.
- $(f \circ g)(x) = f(g(x))$ is not the same as $(f \cdot g)(x) = f(x) \cdot g(x)$.

Composition of functions   Multiplication of functions

Example: Evaluate each expression using the values given in the table.

x	-3	-2	-1	0	1	2	3
$f(x)$	-7	-5	-3	-1	3	5	7
$g(x)$	8	3	0	-1	0	3	8

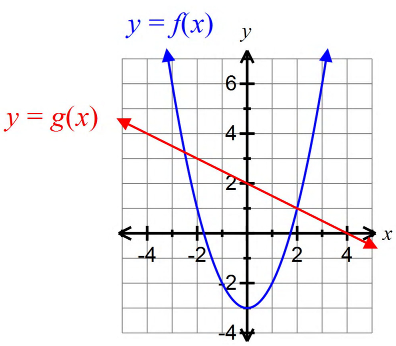
a) $(f \circ g)(-2) = f(g(-2))$
$g(-2) = 3 \quad f(3) = \boxed{7}$

b) $(g \circ f)(-1) = g(f(-1))$
$f(-1) = 0 \quad g(0) = \boxed{-1}$

c) $(f \circ f)(1) = f(f(1))$
$f(1) = 3 \quad f(3) = \boxed{7}$

d) $(g \circ g)(0) = g(g(0))$
$g(0) = -1 \quad g(-1) = \boxed{0}$

Example: Evaluate each expression using the graph.



a) $(f \circ g)(4)$
$g(4) = 0$
$f(0) = \boxed{-3}$

b) $(g \circ f)(-1)$
$f(-1) = -2$
$g(-2) = \boxed{3}$

c) $(f \circ f)(1)$
$f(1) = -2$
$f(-2) = \boxed{1}$

d) $(g \circ g)(0)$
$g(0) = 2$
$g(2) = \boxed{1}$

Example: $f(x) = 2x^2$ and $g(x) = 1 - 3x^2$

a) Find $(f \circ g)(4)$

$g(4) = 1 - 3(4)^2$
$= 1 - 3(16)$
$= 1 - 48$
$= -47$

$f(-47) = 2(-47)^2$
$= 2(2209)$
$= \boxed{4418}$

b) Find $(g \circ f)(2)$

$f(2) = 2(2)^2$
$= 2(4)$
$= 8$

$g(8) = 1 - 3(8)^2$
$= 1 - 3(64)$
$= 1 - 192$
$= \boxed{-191}$

c) Find $(f \circ f)(1)$

$f(1) = 2(1)^2$
$= 2(1)$
$= 2$

$f(2) = 2(2)^2$
$= 2(4)$
$= \boxed{8}$

d) Find $(g \circ g)(0)$

$g(0) = 1 - 3(0)^2$
$= 1 - 3(0)$
$= 1 - 0$
$= 1$

$g(1) = 1 - 3(1)^2$
$= 1 - 3(1)$
$= 1 - 3$
$= \boxed{-2}$

Domain of a Composite Function

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

Example: Find the domain of the composite function $f \circ g$.

a) $f(x) = \dfrac{5}{x+4}$, $g(x) = \dfrac{8}{x}$

$x \to \boxed{\frac{8}{x}} \to g(x) \to \boxed{\frac{5}{x+4}} \to f(g(x))$

$x \neq 0$

$g(x) \neq -4$

$\frac{8}{x} \neq -4$

$8 \neq -4x$

$x \neq -2$

$\{x \mid x \neq 0, -4\}$

b) $f(x) = \dfrac{x}{x-1}$, $g(x) = \dfrac{x+5}{x-4}$

$x \to \boxed{\frac{x+5}{x-4}} \to g(x) \to \boxed{\frac{x}{x-1}} \to f(g(x))$

$x \neq 4$

$g(x) \neq 1$

$\frac{x+5}{x-4} \neq 1$

$x+5 \neq x-4$

$5 \neq -4$

no additional domain restrictions

$\boxed{\{x \mid x \neq 4\}}$

Example: $f(x) = x+1$ and $g(x) = x^2+4$

a) Find $(f \circ g)(x)$ and its domain.

$f(x^2+4)$

$= x^2+4+1 = \boxed{x^2+5}$

b) Find $(g \circ f)(x)$ and its domain.

$g(x+1) = (x+1)^2+4$

$= x^2+2x+1+4$

$= \boxed{x^2+2x+5}$

Example: $f(x) = \dfrac{1}{x+3}$ and $g(x) = -\dfrac{2}{x}$

a) Find $(f \circ g)(x)$ and its domain.

$x \neq 0$ $\quad f\left(-\frac{2}{x}\right) = \dfrac{1}{-\frac{2}{x}+3}$

$= \left(\dfrac{1}{-\frac{2}{x}+3}\right)\left(\dfrac{x}{x}\right)$

$= \boxed{\dfrac{x}{-2+3x}}$ $\quad x \neq 2/3$

$\boxed{\{x \mid x \neq 0, \tfrac{2}{3}\}}$

b) Find $(g \circ f)(x)$ and its domain.

$x \neq -3$ $\quad g\left(\frac{1}{x+3}\right) = -\dfrac{2}{\frac{1}{x+3}}$

$= -2\left(\dfrac{x+3}{1}\right)$

$= \boxed{-2x-6}$

$\boxed{\{x \mid x \neq -3\}}$

c) Find $(f \circ f)(x)$ and its domain.

$x \neq -3$ $\quad f\left(\frac{1}{x+3}\right) = \dfrac{1}{\frac{1}{x+3}+3}$

$= \left(\dfrac{1}{\frac{1}{x+3}+3}\right)\left(\dfrac{x+3}{x+3}\right)$

$= \dfrac{x+3}{1+3(x+3)} = \dfrac{x+3}{1+3x+9}$

$= \boxed{\dfrac{x+3}{3x+10}}$ $\quad x \neq -\frac{10}{3}$

$\boxed{\{x \mid x \neq -3, -\tfrac{10}{3}\}}$

d) Find $(g \circ g)(x)$ and its domain.

$x \neq 0$ $\quad g\left(-\frac{2}{x}\right) = \dfrac{-2}{-\frac{2}{x}}$

$= -2 \cdot -\dfrac{x}{2} = \boxed{x}$

$\boxed{\{x \mid x \neq 0\}}$

Example: Show that $(f \circ g)(x) = (g \circ f)(x) = x$.

a) $f(x) = 4x;\ g(x) = x/4$

$$f(g(x)) = 4\left(\frac{x}{4}\right) = x$$

$$g(f(x)) = \frac{4x}{4} = x$$

b) $f(x) = 4 - 3x;\ g(x) = \frac{1}{3}(4 - x)$

$$\begin{aligned} f(g(x)) &= 4 - 3\left[\tfrac{1}{3}(4-x)\right] \\ &= 4 - (4 - x) \\ &= 4 - 4 + x = x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \frac{1}{3}\left[4 - (4 - 3x)\right] \\ &= \frac{1}{3}(4 - 4 + 3x) \\ &= \frac{1}{3}(3x) = x \end{aligned}$$

Example: Find functions f and g such that $f \circ g = H$.

a) $H(x) = \left(x^2+1\right)^4$

$$x \longrightarrow \underset{\underset{g(x)}{\uparrow}}{\boxed{x^2+1}} \longrightarrow \underset{\underset{f(x)}{\uparrow}}{\boxed{x^4}} \longrightarrow (x^2+1)^4$$

$$\boxed{g(x) = x^2+1 \qquad f(x) = x^4}$$

b) $H(x) = |2x+1|$

$$x \longrightarrow \underset{\underset{g(x)}{\uparrow}}{\boxed{2x+1}} \longrightarrow \underset{\underset{f(x)}{\uparrow}}{\boxed{|x|}} \longrightarrow |2x+1|$$

$$\boxed{g(x) = 2x+1 \qquad f(x) = |x|}$$